Simplified dual-comb hyperspectral digital holography system based on spatial heterodyne interferometry: Supplementary Material.

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Details of the holographic reconstruction algorithm

The signal of interest (SOI) corresponding to the *n*th-order comb line was extracted by applying a spatiotemporal bandpass filter to the beat signal s_{beat}. It is then moved to the low-frequency area to obtain the light field on the detector plane $O_{z,n}$ through the function of:

$$O_{z,n} = E_{pro,n}^{*} E_{ref,n} exp\left[2\pi j\left(-kax - kby - n\Delta f_{rep} t\right)\right]$$

After that, the light field on the object plane O_n can be obtained by back propagating the $O_{z,n}$ with the focal length *z* using the angular spectrum algorithm, which can be expressed as:

$$AS_n = AS_{z,n} exp\left[-j2\pi kz\sqrt{1-(\lambda f_x)^2-\left(\lambda f_y\right)^2}\right]$$

Here, AS_n and $AS_{z,n}$ are the angular spectra of O_n and $O_{z,n}$, respectively, with f_x and f_y denoting the coordinates in the angular spectrum domain. The laser wavelength is represented by λ, and the wavenumber is defined as $k = 1/\lambda$. Finally, O_n can be obtained by applying a 2D inverse fast Fourier transform to AS_n, and then the amplitude map and the phase map of the object can be obtained by calculating the amplitude and argument of O_n, respectively:

$$amplitude = |O_n|, \qquad phase = arctan\left[\frac{Im(O_n)}{Re(O_n)}\right]$$

The above process was repeated for each order of the comb lines, and the amplitude maps and phase maps of the object measured at different wavelengths can then be obtained.

Details of the local phase unwrapping algorithm

With the extension of the depth measurement range, the depth error caused by the phase error will also increase accordingly. In Fig. 3(c), we employed a local phase unwrapping method to address this problem. Specifically, the unwrapped phase map as shown in Fig. 3(b) underwent a thresholding to distinguish the two object surfaces. Subsequently, it was assumed that the absolute phase of the object was locally continuous within each surface. Under this condition, the absolute phase ψ and the wrapped phase φ as shown in Fig. 3(a) satisfies the relation of:

$$\Delta\psi = W[\Delta\varphi] = \begin{cases} \Delta\varphi, |\Delta\varphi| \leq \pi \\ \Delta\varphi - 2\pi, \Delta\varphi > \pi \\ \Delta\varphi + 2\pi, \Delta\varphi < -\pi \end{cases}$$

Then, the absolute phase of a pixel p_1 can be calculated as:

$$\psi(p_1) = \psi(p_0) + \int_L W[\Delta\varphi] dr$$

where p_0 is the starting pixel, L is an integration path linking the two pixels of p_0 and p_1. Since the absolute phase is locally continuous within each surface, p_0, p_1, and the pixels L passes through should belong to the same surface.

This method is essentially an image enhancement algorithm. Based on the priori knowledge that the object was a stepped reflector, the depth error within each reflecting surface was suppressed, so that the distance between the two surfaces can be observed more clearly, as shown in Fig. 3(d).